

# COMPARISON DIFFERENTIAL TRANSFORM METHOD WITH ADOMIAN DECOMPOSITION METHOD FOR NONLINEAR INITIAL VALUE PROBLEMS

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**ABSTRACT.** This paper present a numerical comparison between the differential transformation method (DTM) and Adomian decomposition method (ADM) for solving nonlinear dispersive  $K(m,n)$  equation. In order to show the effectiveness of the DTM, the results obtained from the DTM is compared with available solutions obtained using the ADM and with exact solutions. It illustrates the validity and the great potential of the differential transform method in solving nonlinear partial differential equations.

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**KEYWORDS AND PHRASES.** Nonlinear dispersive  $K(m,n)$  equation, Differential Transform Method, Adomian Decomposition.

## 1. INTRODUCTION

Most phenomena in real world are described through nonlinear equations. Nonlinear phenomena play important roles in applied mathematics, physics and in engineering problems in which each parameter varies depending on different factors. The importance of obtaining the exact or approximate solutions of nonlinear partial differential equations (NLPDEs) in physics and mathematics, it is stil a hot spot to seek now methods to obtain new exact or approximate solutions. In the recent years, many authors mainly had paid attention to study solutions of nonlinear partial differential equations by using various methods. Among these are the Adomian decomposition method (ADM), tanh method, homotopy perturbation method (HPM), sinh-coshmethod, HAM, DTM and variational iteration method (VIM). Since the beginning of the 1980s, Adomian [1]-[4] has presented and developed a so-called decomposition method for solving algebraic, differential, integro-differential equations. The solution is found as an infinite series which converges rapidly to accurate solutions. The method has many advantages over the classical techniques, mainly, it makes unnecessary the linearization, perturbation and other restritive methods and assumptions which may change the problem being solved, sometimes seriously. The concept of differential transform method was first introduced by Zhou [17] in 1986 and it was used to solve both linear and nonlinear initial value problems in electric circuit analysis. The main advantage of this method is that it can be applied directly to NLPDEs without requiring linearization, discretization, or perturbation. It is a semi analytical- numerical technique that formulizes

Taylor series in a very different manner. The method constructs, for differential equations, an analytical solution in the form of a polynomial. Not like the traditional high order Taylor series method that requires symbolic computation, the DTM is an iterative procedure for obtaining Taylor series solutions. Another important advantage is that this method reducing the size of computational work while the Taylor series method is computationally taken long time for large orders. This method is well addressed in [9], [11], [13], [15], [16]. This paper consider the following nonlinear dispersive  $K(m,n)$  equation with fractional time derivatives:

$$(1) \quad u_t + (u^m)_x + (u^n)_{xxx} = 0,$$

where  $m, n > 0$ . The classic nonlinear dispersive  $K(m, n)$  equation first introduced by Rosenau and Hyman [14] and for certain values of  $m$  and  $n$ ,  $K(m, n)$  equation has solitary waves which are compactly supported. Recently, large number of methods were suggested to study the nonlinear dispersive  $K(m, n)$  equations, such as Exp-function method [9], variational iteration method [10], [16], variational method [11] and homotopy perturbation method [10], [12]. In this paper, it is extended the application of the differential transform method to construct analytical approximate solutions of the dispersive  $K[m, n]$  equation (1). Then we compare the results with the previously obtained results by using the ADM in [1]-[4], and exact solutions. With this technique, it is possible to obtain highly accurate results or exact solutions for differential equations.

## 2. DIFFERENTIAL TRANSFORM METHOD

The basic definitions and fundamental operations of the two-dimensional differential transform are defined in [9], [11], [13], [15]. Consider a function of two variable  $w(x, y)$  be analytic in the domain  $\Omega$  and let  $(x, y) = (x_0, y_0)$  in this domain. The function  $w(x, y)$  is then represented by one series whose centre at located at  $w(x_0, y_0)$ . The differential transform of the function is the form

$$(2) \quad W(h, k) = \frac{1}{k!h!} \left[ \frac{\partial^{k+h} w(x, y)}{\partial x^k \partial y^h} \right]_{(x_0, y_0)},$$

where  $w(x, y)$  is the original function and  $W(x, y)$  is the transformed function. The transformation is called T-function and the lower case and upper case letters represent the original and transformed functions respectively. Then its inverse transform is defined as

$$(3) \quad w(x, y) = \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} W(x, y) (x - x_0)^k (y - y_0)^h$$

The relations 2 and 3 imply that

$$(4) \quad w(x, y) = \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} \frac{1}{k!h!} \left[ \frac{\partial^{k+h} w(x, y)}{\partial x^k \partial y^h} \right]_{(x_0, y_0)} (x - x_0)^k (y - y_0)^h$$

In a real application and when  $(x_0, y_0)$  are taken as  $(0, 0)$ , then the function  $w(x, y)$  is expressed by a finite series and Eq.(3) can be written as

$$(5) \quad w(x, y) \cong \sum_{k=0}^m \sum_{h=0}^n W(x, y)x^k y^h,$$

in addition, Eq.(5) implies that  $\sum_{k=m+1}^{\infty} \sum_{h=n+1}^{\infty} W(x, y)x^k y^h$  is negligibly small. Usually, the values of  $m$  and  $n$  are decided by convergences of the series coefficients.

Table 1: Operations for the two-dimensional differential transformation

Original function	Transformed function
$w(x, y) = u(x, y) \pm v(x, y)$	$W(k, h) = U(k, h) \pm V(k, h)$
$w(x, y) = \alpha u(x, y)$	$W(k, h) = \alpha U(k, h)$
$w(x, y) = \frac{\partial u(x, y)}{\partial x}$	$W(k, h) = (k + 1)U(k + 1, h)$
$w(x, y) = \frac{\partial u(x, y)}{\partial y}$	$W(k, h) = (h + 1)U(k, h + 1)$
$w(x, y) = \frac{\partial u^{r+s}(x, y)}{\partial x^r \partial y^s}$	$W(k, h) = (k + 1)(k + 2) \cdots (k + r)(h + 1)(h + 2) \cdots (h + s)U(k + r, h + s)$
$w(x, y) = u(x, y)v(x, y)$	$W(k, h) = \sum_{r=0}^k \sum_{s=0}^h U(r, h - s)V(k - r, s)$
$w(x, y) = x^m y^n$	$W(k, h) = \delta(k - m, h - n) = \delta(k - m)\delta(h - n) = \begin{cases} 1, & \text{for } k = m \text{ and } h = n \\ 0, & \text{otherwise} \end{cases}$
$w(x, y) = u(x, y)v(x, y)\frac{\partial c^2(x, y)}{\partial x^2}$	$W(k, h) = \sum_{r=0}^k \sum_{t=0}^{k-r} \sum_{s=0}^h \sum_{p=0}^{h-s} (k - r - t + 2)(k - r - t + 1)U(r, h - s - p)V(t, s)C(k - r - t + 2, p)$
$w(x, y) = \frac{\partial u(x, y)}{\partial x} \frac{\partial v(x, y)}{\partial x}$	$W(k, h) = \sum_{r=0}^k \sum_{s=0}^h (r + 1)(k - r + 1)U(r + 1, h - s)V(k - r + 1, s)$
$w(x, y) = u(x, y)\frac{\partial v^2(x, y)}{\partial x^2}$	$W(k, h) = \sum_{r=0}^k \sum_{s=0}^h (k - r + 2)(k - r + 1)U(r, h - s)V(k - r + 2, s)$
$w(x, y) = u(x, y)v(x, y)q(x, y)$	$W(k, h) = \sum_{r=0}^k \sum_{t=0}^{k-r} \sum_{s=0}^h \sum_{p=0}^{h-s} U(r, h - s - p)V(t, s)Q(k - r - t, p)$

### 3. ADOMIAN DECOMPOSITION METHOD

Consider the differential equation

$$(6) \quad Lu + Ru + Nu = 0,$$

where  $L$  and  $R$  are linear differential operators, and  $Nu$  represents the nonlinear terms. The operator  $L$  is assumed to be easily invertible. Applying the inverse operator  $L^{-1}$  to both sides of (6), and using the given conditions we obtain

$$(7) \quad u = f - L^{-1}(Ru) - L^{-1}(Nu),$$

where  $f$  is the function that arises from the given initial conditions that are assumed to be prescribed. Adomian decomposition method defines the solution  $u$  by a series of components

$$(8) \quad U = \sum_{n=0}^{\infty} u_n,$$

where the components  $u_0, u_1, u_2, \dots$  are usually determined recursively by using the relation

$$(9) \quad u_0 = f, \quad u_{k+1} = -L^{-1}(Ru_k) - L^{-1}(Nu_k), \quad k \geq 0.$$

The nonlinear term  $F(u) = Nu$  is represented by an infinite series

$$(10) \quad F(u) = \sum_{n=0}^{\infty} A_n,$$

where  $A_n$  are the so-called Adomian polynomials that can be calculated for all forms of nonlinearity according to algorithms set by Adomian [3] defined by

$$(11) \quad A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[ F \left( \sum_{i=0}^{\infty} \lambda^i u_i \right) \right]_{\lambda=0},$$

$n = 1, 2, 3, \dots$

#### 4. ILLUSTRATIVE EXAMPLES

In this section, we have chosen to present two test problems, namely  $K(2, 2)$  and  $K(3, 3)$  with new initial conditions to be considered.

**Example 4.1.** We first consider the initial value problem  $K(2, 2)$

$$(12) \quad u_t + (u^2)_x + (u^2)_{xxx} = 0, \quad u(x, 0) = \frac{4}{3}c \sin^2 \left( \frac{1}{4}x \right)$$

with the exact solution [5]

$$(13) \quad u(x, t) = \frac{4}{3}c \sin^2 \left( \frac{x - ct}{4} \right)$$

Taking the two-dimensional transform of Eq. (12) by using the related definitions in Table 1, we have

$$(14) \quad u(k, h+1) = \frac{\left( \begin{array}{l} -2 \left( \sum_{r=0}^k \left( \sum_{s=0}^h (k-r+1) u(r, h-s) u(k-r+1, s) \right) \right) \\ -6 \left( \sum_{r=0}^k \left( \sum_{s=0}^h (k-r+1)(k-r+2)(r+1) u(r+1, h-s) u(k-r+2, s) \right) \right) \\ -2 \left( \sum_{r=0}^k \left( \sum_{s=0}^h (k-r+1)(k-r+2)(k-r+3) u(r, h-s) u(k-r+3, s) \right) \right) \end{array} \right)}{(h+1)}.$$

by applying the differential transform into (12), the initial transformation coefficients are thus determined by

$$\begin{aligned}
(15) \quad u(x, t) = & \frac{1}{12}c^3t^2 - \frac{1}{576}c^5t^4 - \frac{1}{6}c^2xt + \frac{1}{144}c^4xt^3 - \frac{1}{11520}c^6xt^5 \\
& + \frac{1}{12}cx^2 - \frac{1}{96}c^3x^2t^2 + \frac{1}{4608}c^5x^2t^4 + \frac{1}{144}c^2x^3t - \frac{1}{3456}c^4x^3t^3 \\
& + \frac{1}{276480}c^6x^3t^5 - \frac{1}{576}cx^4 + \frac{1}{4608}c^3x^4t^2 - \frac{1}{221184}c^5x^4t^4 \\
& - \frac{1}{11520}c^2x^5t + \frac{1}{276480}c^4x^5t^3 - \frac{1}{22118400}c^6x^5t^5
\end{aligned}$$

Hence from Eq. (15)  $U(k, 0) = 0$ , if  $k = 1, 3, 5, \dots$

$$\begin{aligned}
(16) \quad U(2, 0) &= \frac{c}{12} \\
U(4, 0) &= -\frac{c}{576} \\
U(6, 0) &= -\frac{c}{6}, \dots
\end{aligned}$$

Substituting Eq. (16) in Eq. (14), and by recursive method we can calculate others values of  $U(h, k)$ . Our approximation has one more interesting property, if we expand exact solution (13) using Taylor expansion about  $(0, 0)$ , we have the series same as the our approximation (15). Now, we consider ADM for same equation.

Applying  $L^{-1}$  to both sides of (12) yields

$$(17) \quad u(x, t) = \frac{4}{3}c \sin^2\left(\frac{1}{4}x\right) - L^{-1}((u^2)_x + (u^2)_{xx}).$$

Substituting the decomposition series (8) for  $u(x, t)$  into (17) gives

$$(18) \quad \sum_{n=0}^{\infty} u_n(x, t) = \frac{4}{3}c \sin^2\left(\frac{1}{4}x\right) - L^{-1}\left(\sum_{n=0}^{\infty} A_n + \sum_{n=0}^{\infty} B_n\right),$$

where  $A_n$  and  $B_n$  are Adomian Polynomials that represent the nonlinear operators  $(u^2)_x$  and  $(u^2)_{xx}$ , respectively. Following our discussion above, we introduce the recursive relation

$$(19) \quad u_0(x, t) = \frac{4}{3}c \sin^2\left(\frac{1}{4}x\right), \quad u_{k+1}(x, t) = -L^{-1}(A_k + B_k), \quad k \geq 0.$$

To calculate Adomian polynomials  $A_n$  and  $B_n$ , we substitute  $m = n = 2$  in (11) to obtain:

$$\begin{aligned}
(20) \quad A_0 &= F(u_0) = (u_0^2)_x, \\
A_1 &= u_1 F'(u_0) \\
&= (2u_1 u_0)_x \\
A_2 &= u_2 F'(u_0) + \frac{1}{2}u_1^2 F''(u_0) \\
&= (2u_2 u_0 + u_1^2)_x
\end{aligned}$$

and

$$\begin{aligned}
 B_0 &= G(u_0) \\
 &= (u_0)_{xxx}^2, \\
 B_1 &= u_1 G'(u_0) \\
 &= (2u_1 u_0)_{xxx}, \\
 B_2 &= u_2 G'(u_0) + \frac{1}{2} u_1^2 G''(u_0) \\
 (21) \quad &= (2u_2 u_0 + u_1^2)_{xxx},
 \end{aligned}$$

substituting (20) and (21) into (19) gives:

$$\begin{aligned}
 u_0(x, t) &= \frac{4}{3} c \sin^2 \left( \frac{1}{4} x \right), \\
 u_1(x, t) &= -L^{-1}(A_0 + B_0) \\
 &= -\frac{1}{3} c^2 t \sin \left( \frac{1}{4} x \right), \\
 (22) \quad u_2(x, t) &= -L^{-1}(A_1 + B_1) \\
 &= \frac{1}{12} c^3 t^2 \cos \left( \frac{1}{2} x \right), \\
 u_3(x, t) &= -L^{-1}(A_2 + B_2) \\
 &= \frac{1}{72} c^4 t^3 \sin \left( \frac{1}{2} x \right), \\
 &\vdots \\
 &\vdots \\
 &\vdots
 \end{aligned}$$

This gives the solution in a series form

$$\begin{aligned}
 (23) \quad u(x, t) &= \frac{4}{3} c \sin^2 \left( \frac{1}{4} x \right) - \frac{1}{3} c^2 t \sin \left( \frac{1}{4} x \right) + \frac{1}{12} c^3 t^2 \cos \left( \frac{1}{2} x \right) \\
 &+ \frac{1}{72} c^4 t^3 \sin \left( \frac{1}{2} x \right) + \dots
 \end{aligned}$$

**Example 4.2.** We now consider the initial value problem  $K(3, 3)$

$$(24) \quad u_t + (u^3)_x + (u^3)_{xxx} = 0, \quad u(x, 0) = \frac{\sqrt{6c}}{2} \sin \left( \frac{1}{3} x \right)$$

with the exact solution

$$(25) \quad u(x, t) = \frac{\sqrt{6c}}{2} \sin \left( \frac{x - ct}{3} \right)$$

Taking the two-dimensional transform of Eq. (24) by using the related definitions in Table 1, we have

$$(26) \quad u(k, h+1) = \frac{\left( \begin{aligned} & -3 \left( \sum_{r=0}^k \left( \sum_{t=0}^{k-r} \left( \sum_{s=0}^h \left( \sum_{p=0}^{h-s} (k-r-t+1)u(r, h-s-p)u(t, s)u(k-r-t+1, p) \right) \right) \right) \right) \right) \\ & -6 \left( \sum_{r=0}^k \left( \sum_{t=0}^{k-r} \left( \sum_{s=0}^h \left( \sum_{p=0}^{h-s} (k-r-t+1)(r+1)(t+1)u(r+1, h-s-p)u(t+1, s)u(k-r-t+1, p) \right) \right) \right) \right) \\ & -18 \left( \sum_{r=0}^k \left( \sum_{t=0}^{k-r} \left( \sum_{s=0}^h \left( \sum_{p=0}^{h-s} (k-r-t+1)(k-r-t+2)(t+1)u(r, h-s-p)u(t+1, s)u(k-r-t+2, p) \right) \right) \right) \right) \\ & -3 \left( \sum_{r=0}^k \left( \sum_{t=0}^{k-r} \left( \sum_{s=0}^h \left( \sum_{p=0}^{h-s} (k-r-t+1)(k-r-t+2)(k-r-t+3)u(r, h-s-p)u(t, s)u(k-r-t+3, p) \right) \right) \right) \right) \end{aligned} \right)}{(h+1)}$$

by applying the differential transform into (25), the initial transformation coefficients are thus determined by

$$(27) \quad \begin{aligned} u(x, t) = & -\frac{1}{6}\sqrt{6}c^{3/2}t + \frac{1}{324}\sqrt{6}c^{7/2}t^3 + \frac{1}{6}\sqrt{6}\sqrt{c}x - \frac{1}{108}\sqrt{6}c^{5/2}xt^2 \\ & + \frac{1}{108}\sqrt{6}c^{3/2}x^2t - \frac{1}{5832}\sqrt{6}c^{7/2}x^2t^3 - \frac{1}{324}\sqrt{6}\sqrt{c}x^3 \\ & + \frac{1}{5832}\sqrt{6}c^{5/2}x^3t^2 \end{aligned}$$

Hence from Eq.(27)

$$(28) \quad \begin{aligned} u(0, 0) &= 0, \quad k = 0, 2, 4, \dots \\ u(1, 0) &= \frac{\sqrt{6c}}{6} \\ u(3, 0) &= -\frac{\sqrt{6c}}{324} \\ u(0, 1) &= -\frac{\sqrt{6}}{6}c^{3/2} \\ u(0, 3) &= -\frac{\sqrt{6}}{324}c^{7/2} \\ u(1, 2) &= -\frac{\sqrt{6}}{108}c^{5/2} \\ u(3, 2) &= -\frac{\sqrt{6}}{5832}c^{5/2} \end{aligned}$$

Substituting Eq. (28) in Eq. (26), and by recursive method we can calculating another values of  $U(h, k)$ .

If we expand exact solution (24) using Taylors expansion about (0, 0), we have the series same as the our approximation (27).

We consider ADM for same equation. In a paralel manner to our analysis presented above we obtain

$$(29) \quad u(x, t) = \frac{\sqrt{6c}}{2} \sin\left(\frac{1}{3}x\right) - L^{-1}((u^3)_x + (u^3)_{xxx}).$$

Using the decomposition series assumption (8) for  $u(x, t)$  gives

$$(30) \quad \sum_{n=0}^{\infty} u_n(x, t) = \frac{\sqrt{6c}}{2} \sin\left(\frac{1}{3}x\right) - L^{-1}\left(\sum_{n=0}^{\infty} A_n^{\%} + \sum_{n=0}^{\infty} B_n^{\%}\right),$$

where  $A_n^{\%}$  and  $B_n^{\%}$  are Adomian polynomials that represent the nonlinear operators  $(u^3)_x$  and  $(u^3)_{xxx}$ , respectively. In view of (30), we use the recursive relation

$$(31) \quad u_0(x, t) = \frac{\sqrt{6c}}{2} \sin\left(\frac{1}{3}x\right), \quad u_{k+1}(x, t) = -L^{-1}(A_k^{\%} + B_k^{\%}), \quad k \geq 0.$$

Adomian polynomials  $A_n^{\%}$  and  $B_n^{\%}$  can be calculated as before to find:

$$(32) \quad \begin{aligned} A_0^{\%} &= (u_0^3)_x, \\ A_1^{\%} &= (3u_1u_0^2)_x, \\ A_2^{\%} &= (3u_2u_0^2 + 3u_0u_1^2)_x, \\ &\vdots \\ &\vdots \\ &\vdots \end{aligned}$$

and

$$(33) \quad \begin{aligned} B_0^{\%} &= (u_0^3)_{xxx}, \\ B_1^{\%} &= (3u_1u_0^2)_{xxx}, \\ B_2^{\%} &= (3u_2u_0^2 + 3u_0u_1^2)_{xxx}, \end{aligned}$$

Substituting (32) and (33) into (31) gives:

$$(34) \quad \begin{aligned} u_0(x, t) &= \frac{\sqrt{6c}}{2} \sin\left(\frac{1}{3}x\right), \\ u_1(x, t) &= -L^{-1}(A_0 + B_0) \\ &= -\frac{\sqrt{6c^3}}{6} t \cos\left(\frac{1}{3}x\right), \\ u_2(x, t) &= -L^{-1}(A_1 + B_1) \\ &= -\frac{\sqrt{6c^5}}{36} t^2 \sin\left(\frac{1}{3}x\right), \\ u_3(x, t) &= -L^{-1}(A_2 + B_2) \\ &= -\frac{\sqrt{6c^7}}{324} t^3 \cos\left(\frac{1}{3}x\right), \end{aligned}$$

Consequently, the solution in a series form is given by

$$(35) \quad u(x, t) = \frac{\sqrt{6c}}{2} \sin\left(\frac{1}{3}x\right) - \frac{\sqrt{6c^3}}{6} t \cos\left(\frac{1}{3}x\right) - \frac{\sqrt{6c^5}}{36} t^2 \sin\left(\frac{1}{3}x\right) + \frac{\sqrt{6c^7}}{324} t^3 \cos\left(\frac{1}{3}x\right) + \dots$$



## 5. CONCLUSION

This paper applied the differential transformation technique and Adomian decomposition method to solve initial value problem. Some difficulties in the Adomian decomposition methods disappear by DTM. Differential transform method is equivalent to the Adomian decomposition method in this problem. As an advantage of the differential transform method over the decomposition procedure of Adomian, DTM provides a solution to the problem without calculating Adomian polynomials. In this work, we use the MAPLE Package to calculate the series obtained from the methods.

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